

Topology

Ques - Find the closure of the set $\left\{ 2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \dots \right\}$ with respect to usual topology on \mathbb{R} .

Ans. Let $A = \left\{ 2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \dots \right\}$

Then A is expressible as

$$\begin{aligned} A &= \left\{ 1+1, 1+\frac{1}{2}, 1+\frac{1}{3}, 1+\frac{1}{4}, 1+\frac{1}{5}, \dots \right\} \\ &= \left\{ 1+\frac{1}{n} : n \in \mathbb{N} \right\} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(1+\frac{1}{n} \right) = 1$$

$$\therefore D(A) = \{1\}$$

$$\bar{A} = A \cup D(A)$$

$$= \{1\} \cup A$$

$$= \left\{ 1, 2, \frac{3}{2}, \frac{4}{3}, \frac{4}{5}, \frac{6}{5}, \dots \right\}$$

$$= \left\{ 1, 2, \frac{3}{2}, \frac{4}{3}, \frac{4}{5}, \frac{6}{5}, \dots \right\}$$

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